Roll No. ....

24018

B. Tech. 4th (Common for all Branches) Semester (Re-Appear) Examination – October, 2020

MATHEMATICS-II

Paper : Math-102

Time : 1.45 hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt any *three* questions. All questions carry equal marks.

1. (a) Verify the formula $\frac{d}{dt}(\vec{A}.\vec{B}) = \vec{A}.\frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt}.\vec{B}$ for $\vec{A} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$, $\vec{B} = \sin t\hat{i} - \cos t\hat{j}$.

   (b) State Stoke's theorem.

   (c) Find the orthogonal trajectories of hyperbola $xy = c$.

   (d) Solve $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$, given that when $t = 0$, $x = 0$ and $\frac{dx}{dt} = 0$.

   (e) Find :

$$L\left[e^{-t}(\sin 2t - 2t\cos 2t)\right]$$

   (f) Find the Laplace transform of the square wave function of period a defined as :

$$f(t) = 1, \text{ when } 0 < t < \frac{a}{2}$$
$$= -1, \text{ when } \frac{a}{2} < t < a$$

   (g) Solve :

$$p^2 - q^2 = x - y$$

   (h) Solve the following equation by method of separation of variables :

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(x,0) = 4e^{-x}$$

2. (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - z$ at the point (2, –1, 2).

   (b) Find the values of a, b, c for which vector $\vec{V} = (x + y + az)\hat{i} + (bx + 3y - z)\hat{j} + (3x + cy + z)\hat{k}$ is irrotational.

3. (a) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

(b) Find $\iint_S \vec{F}.\hat{n}dS$, where $\vec{F} = (2x+3z)\hat{i} + (xz+y)\hat{j} + (y^2+2z)\hat{k}$ and S is the surface of the sphere having centre at (3, –1, 2) and radius 3.

4. (a) Solve :

$$(3xy - 2ay^2)dx + (x^2 - 2axy)dy = 0$$

(b) If the temperature of the air is 20°C and body cools from 100°C to 60°C in 20 minutes, in what time will temperature drop to 30°C ? What will be the temperature of the body after 40 minutes ?

5. (a) Solve :

$$\frac{d^2y}{dx^2} + y = x\sin x$$

by method of variation of parameters.

(b) A second's pendulum which gains 10 seconds per day at one place loses 10 seconds per day at another; compare the accelerations due to gravity at the two places.

6. (a) (i) Find :

$$L\left[\frac{1-\cos t}{t^2}\right]$$

(ii) Evaluate :

$$\int_0^\infty te^{-2t}\cos t\,dt$$

(b) State and prove Convolution theorem.

7. (a) Solve the following simultaneous equation by using L.T. $\frac{dx}{dt} - 2x + 3y = 0, \frac{dy}{dt} + 2x - y = 0$ given that $x(0) = 8$ and $y(0) = 3$.

(b) Solve the integral equation :

$$\int_0^t \frac{y(u)}{\sqrt{t-u}}du = 1 + t + t^2$$

8. (a) Form partial differential equation by eliminating the arbitrary function form $f(x^2+y^2+z^2, z^2-2xy) = 0$.

(b) Solve :

$$(y+z)p + (z+x)q = x+y$$

9. (a) Solve :

$$2xz - px^2 - 2qxy + pq = 0$$

(b) Solve :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

which satisfies the conditions : $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\frac{n\pi x}{l}$.